B   **SHAPE AND SPACE**

Transformations

— *Transformations*

Clarification: Students need to be familiar with the term "invariant points" as points that are not altered by a transformation.

B1   *describe how vertical and horizontal translations of functions affect graphs and their related equations:*

$$y = f(x - h)$$
$$y - k = f(x)$$

Knowledge
B1

1. If the graph of $2x + 3y = 5$ is translated 4 units up, determine an equation of the new graph.

   A. $2x + 3y = 1$

   B. $2x + 3y = 9$

   C. $2x + 3(y + 4) = 5$

   D. $2x + 3(y - 4) = 5$

Understanding
B1

2. If (a, b) is a point on the graph of $y = f(x)$, determine a point on the graph of $y = f(x - 2) + 3$.

   A. $(a - 2, b + 3)$

   B. $(a - 2, b - 3)$

   C. $(a + 2, b + 3)$

   D. $(a + 2, b - 3)$

3. If the point $(2, -8)$ is on the graph of $y = f(x-3)+4$, what point must be on the graph of $y = f(x)$?

   A. $(-1, -12)$

   B. $(-1, -4)$

   C. $(5, -12)$

   D. $(5, -4)$

B2 *describe how compressions and expansions of functions affect graphs and their related equations:*

$$y = af(x)$$
$$y = f(kx)$$

Knowledge

B2

4. How is the graph of $y = 7^{3x}$ related to the graph of $y = 7^x$?

    A. The graph of $y = 7^x$ has been expanded vertically by a factor of 3.

    B. The graph of $y = 7^x$ has been compressed vertically by a factor of $\dfrac{1}{3}$.

    C. The graph of $y = 7^x$ has been expanded horizontally by a factor of 3.

    D. The graph of $y = 7^x$ has been compressed horizontally by a factor of $\dfrac{1}{3}$.
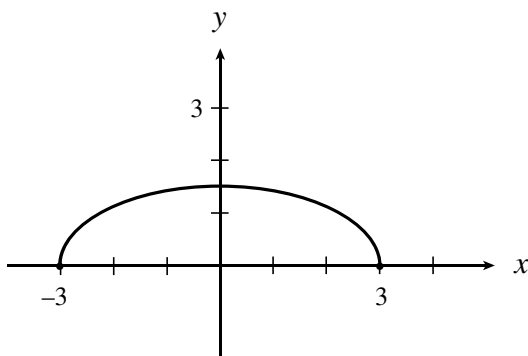
B2

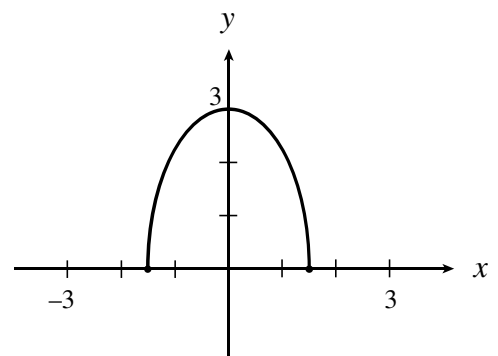5. The graph of $y = \sqrt{9 - x^2}$ is shown below.
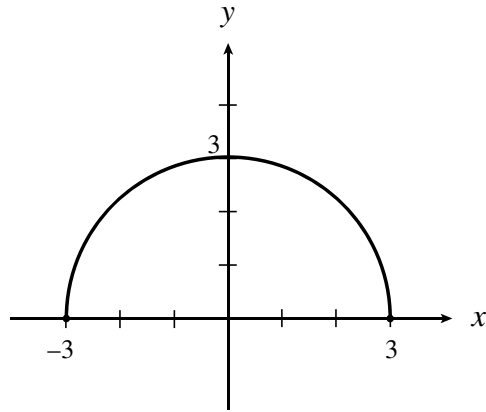
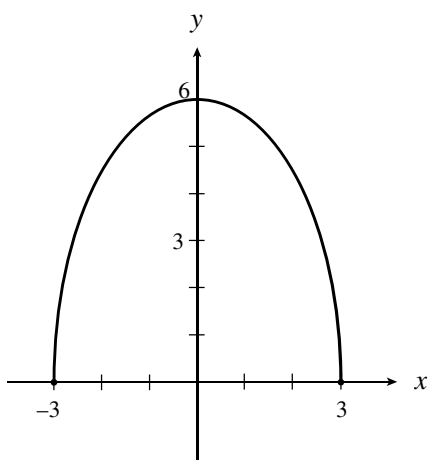Which of the following graphs represents $2y = \sqrt{9 - x^2}$?

A.
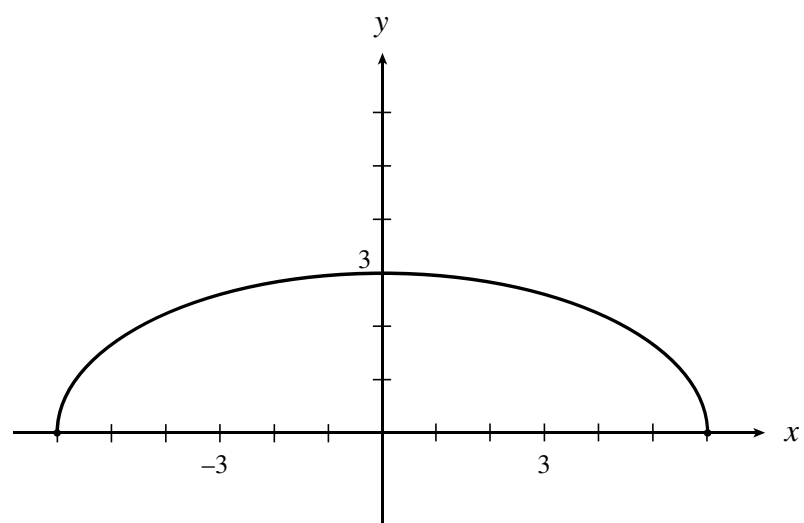

B.

C.

D.

6. If the graph of $x^2 + y^2 = 4$ is vertically compressed by a factor of $\dfrac{1}{5}$, then reflected in the y-axis, determine an equation for the new graph.

A. $x^2 + \dfrac{y^2}{25} = 4$

B. $x^2 + 25y^2 = 4$

C. $-x^2 + 25y^2 = 4$

D. $-x^2 + \dfrac{y^2}{25} = 4$

B3 *describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations:*

$$y = f(-x) \qquad\qquad y = -f(x) \qquad\qquad y = f^{-1}(x)$$

Knowledge
B3

7. The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in

    A. the y-axis.
    B. the x-axis.
    C. the line $y = x$.
    D. the line $y = -x$.

Knowledge
B3

8. What is the inverse of the relation $y = x^3$?

    A. $y = \dfrac{1}{x^3}$

    B. $x = y^3$

    C. $y = (-x)^3$

    D. $x = y^{\frac{1}{3}}$

9. The point $(6, -12)$ is on the graph of the function $y = f(x)$. Which point must be on the graph of the function $y = 3f(-x)$?

   A. $(-6, -36)$
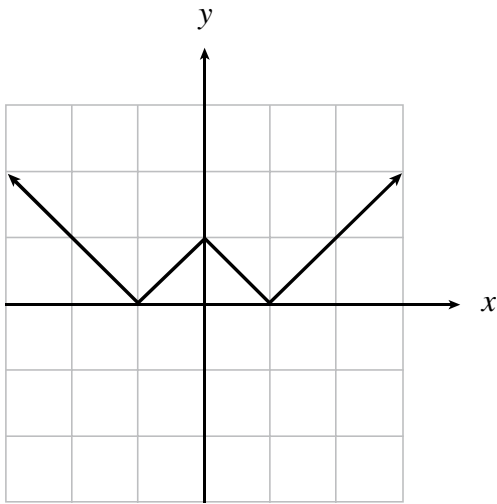   B. $(6, 36)$
   C. $(-6, -4)$
   D. $(6, 4)$

10. If $f(x) = \dfrac{2x}{x-1}$, determine the equation of $f^{-1}(x)$, the inverse of $f(x)$.

   A. $f^{-1}(x) = \dfrac{x}{x-2}$

   B. $f^{-1}(x) = \dfrac{2x}{2x-1}$

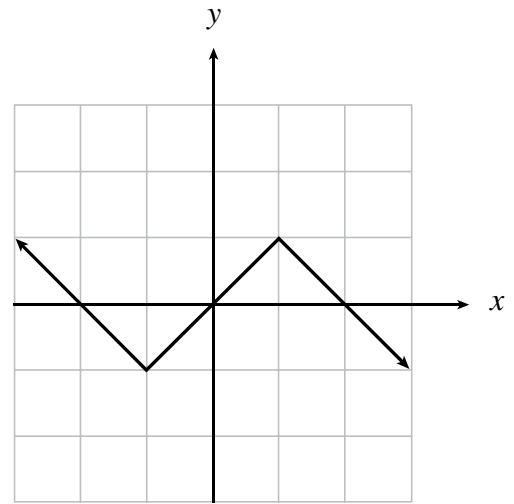   C. $f^{-1}(x) = \dfrac{x-1}{2x}$

   D. $f^{-1}(x) = \dfrac{1}{x-2}$

11. For which graph of $y = f(x)$ would $f(-x) = -f(x)$?

A.

B.

C.

D.
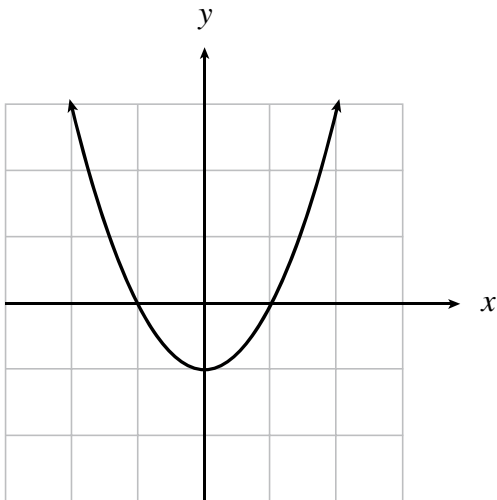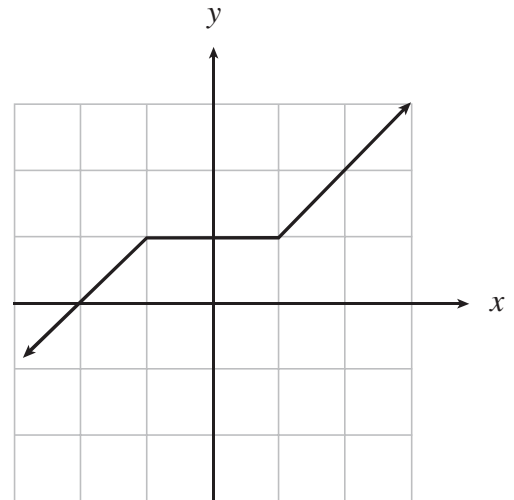
12. When the graph of $y = f(x)$ is transformed to the graph of $y = f(-x)$, on which line(s) will the invariant points lie?

   A. $y = 0$
   B. $x = 0$
   C. $y = x$
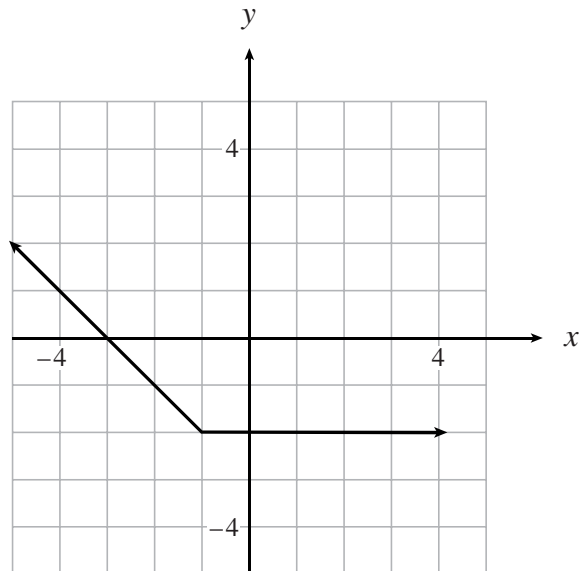   D. $y = 1, \ y = -1$

Knowledge

B4

13. Given the graph of $y = f(x)$ below, determine an equation for an asymptote for the graph of $y = \dfrac{1}{f(x)}$.
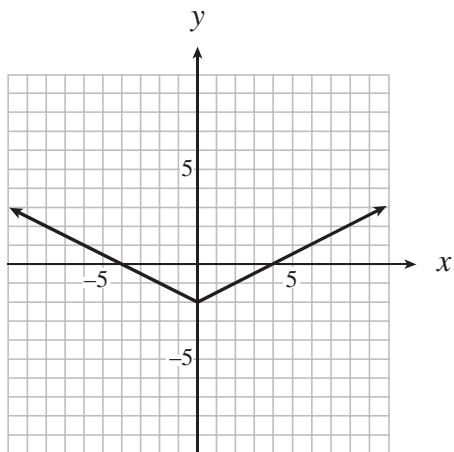

   A. $x = 3$
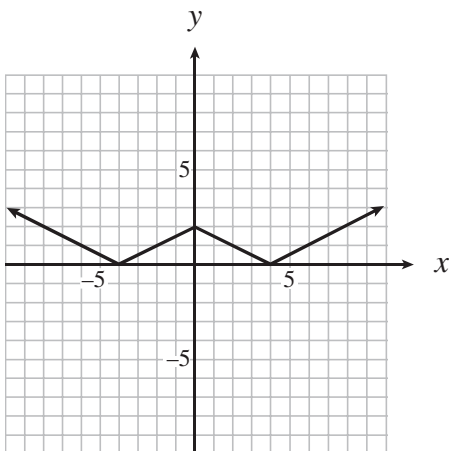   B. $x = -3$
   C. $y = -2$
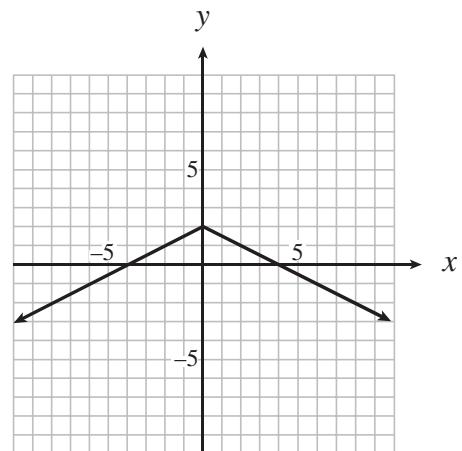   D. $y = 2$

14. The graph of $y = f(x)$ is shown below.

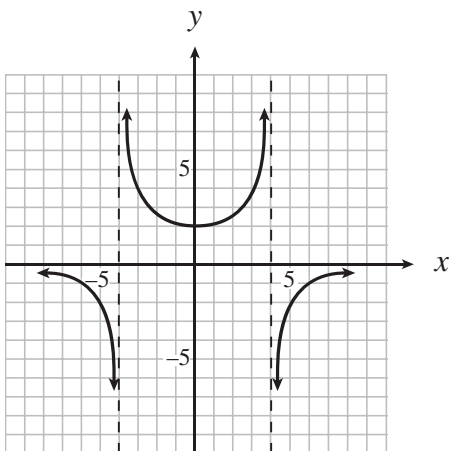Which of the following graphs represents $y = \dfrac{1}{f(x)}$?

A.
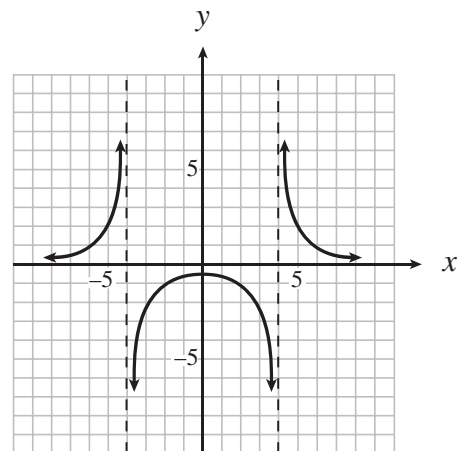

B.

C.

D.

15. If the range of $y = f(x)$ is $-1 \le y \le 2$, what is the range of $y = \dfrac{1}{f(x)}$?

A. $-1 \le y \le \dfrac{1}{2}$

B. $-1 \le y \le \dfrac{1}{2}$, $y \ne 0$

C. $y \ge \dfrac{1}{2}$ or $y \le -1$
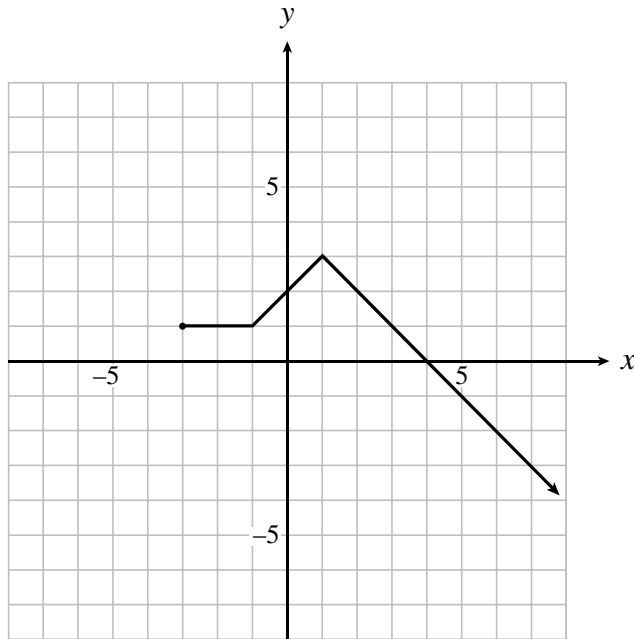
D. $y \ge 2$ or $y \le -1$

Understanding

B4

16. The graph of $y = f(x)$ is transformed to the graph of $y = \dfrac{1}{f(x)}$. If the following points are on the graph of $y = f(x)$, which point would be invariant?

A. $(1, 2)$
B. $(2, 1)$
C. $(3, 0)$
D. $(0, 3)$

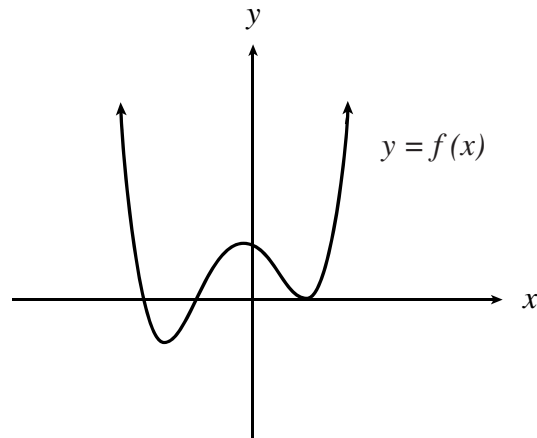17. The graph of $y = f(x)$ is shown below.

On the grid provided, sketch the graph of $y = \dfrac{1}{f(x)}$.

B5 *using the graph and/or the equation of $f(x)$, describe and sketch $|f(x)|$*
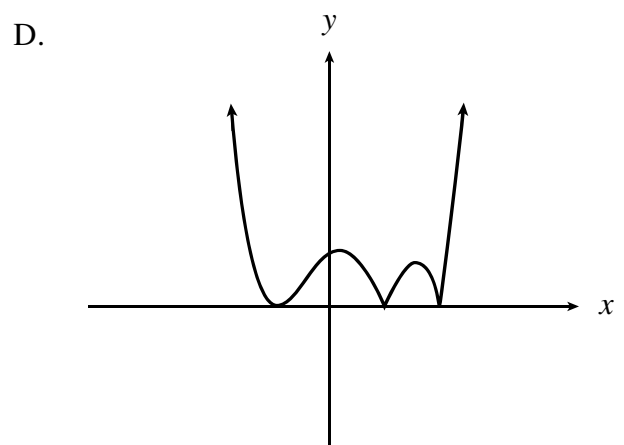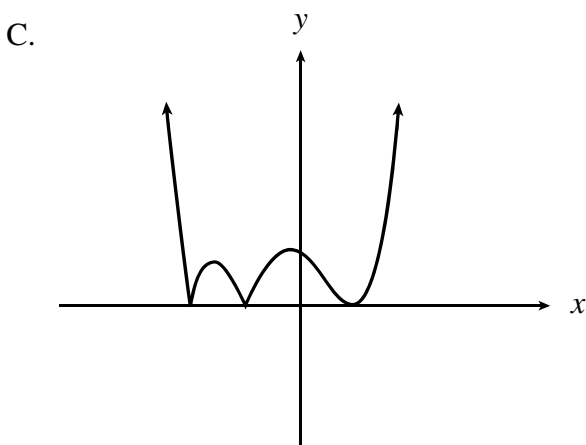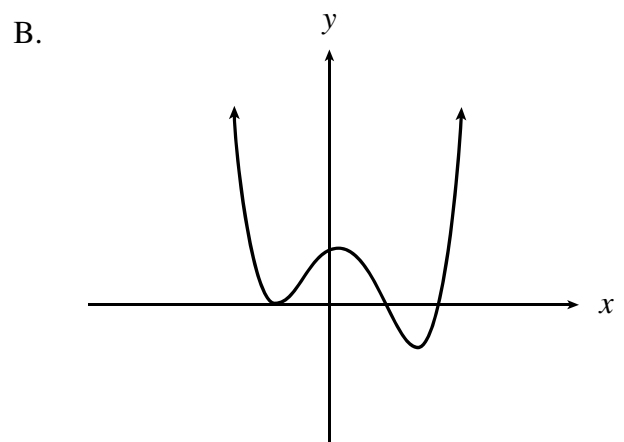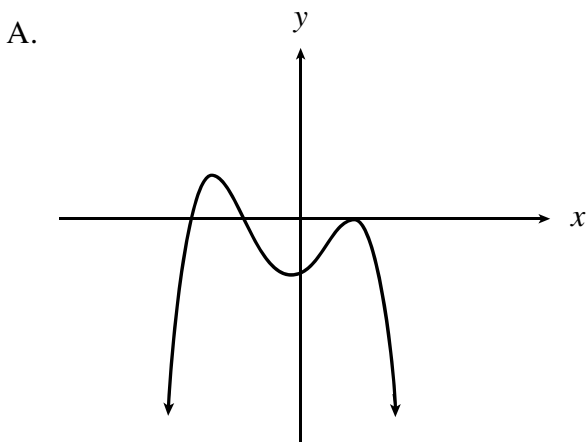
Understanding

B5

18. The graph of the function $y = f(x)$ is shown below.

Which of the following is a graph of $y = |f(x)|$?

A.

B.

C.

D.

19. If the range of $y = f(x)$ is $-3 \le y \le 5$, what is the range of $y = |f(x)|$?

    A. $-3 \le y \le 5$
    B. $0 \le y \le 3$
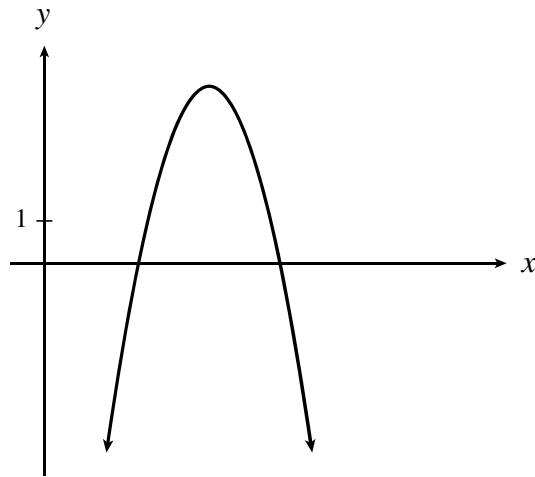   C. $0 \le y \le 5$
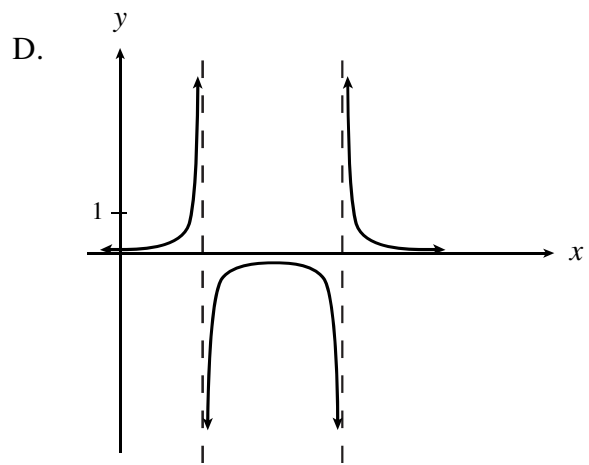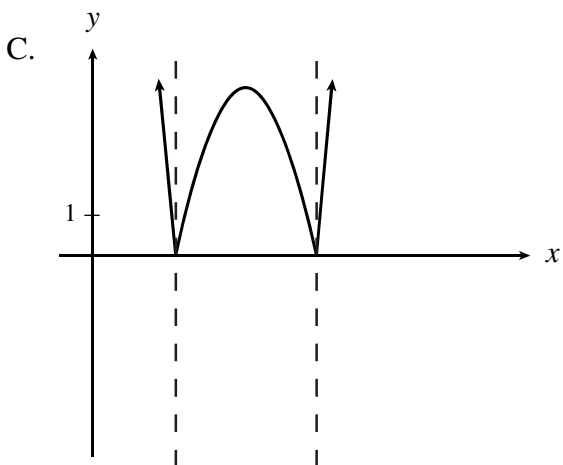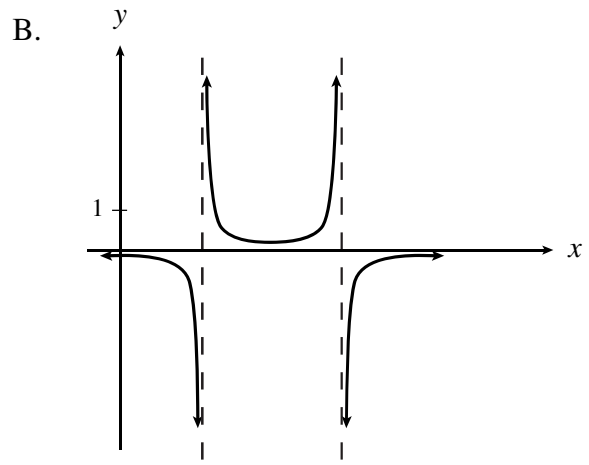   D. $3 \le y \le 5$
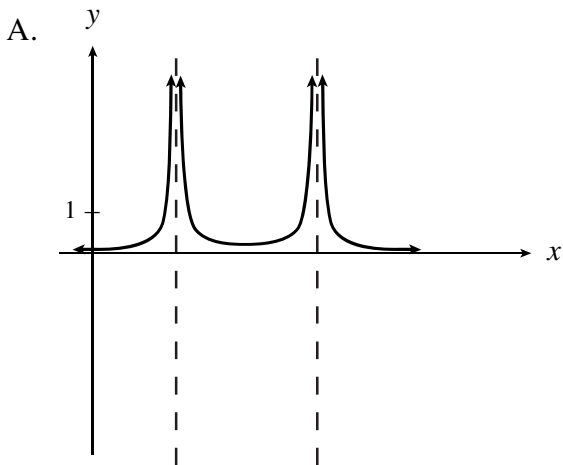
20. The graph of the function $y = f(x)$ is shown below.

Which of the following is a graph of $y = \dfrac{1}{|f(x)|}$?

A.

B.

C.

D.

B6 *describe and perform single transformations and combinations of transformations on functions and relations*

Clarification: The absolute value of a function and the reciprocal value of a function may also be combined with transformations.

Knowledge

B6

21. Determine an equation that will cause the graph of $y = f(x)$ to expand vertically by a factor of 4 and shift 3 units up.

A. $y = \dfrac{1}{4} f(x) + 3$

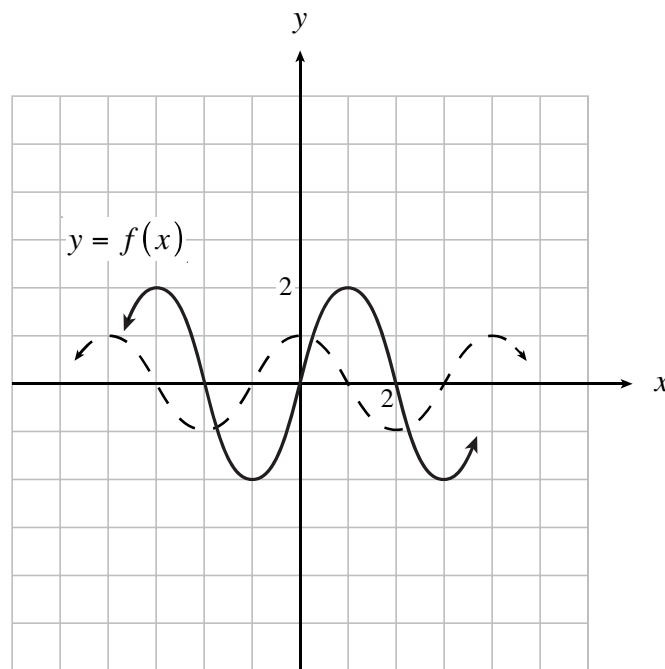B. $y = \dfrac{1}{4} f(x) - 3$

C. $y = 4 f(x) + 3$

D. $y = 4 f(x) - 3$

22. In the diagram below, $y = f(x)$ is graphed as a broken line.

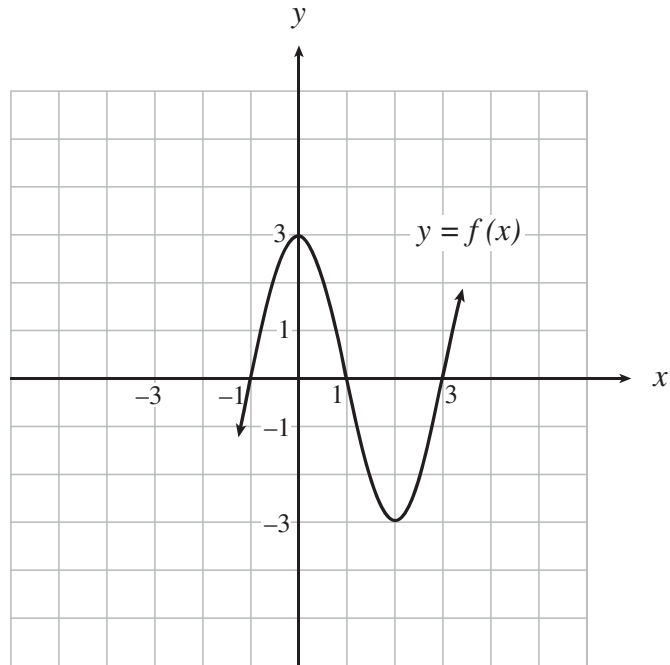Which equation is defined by the solid line?

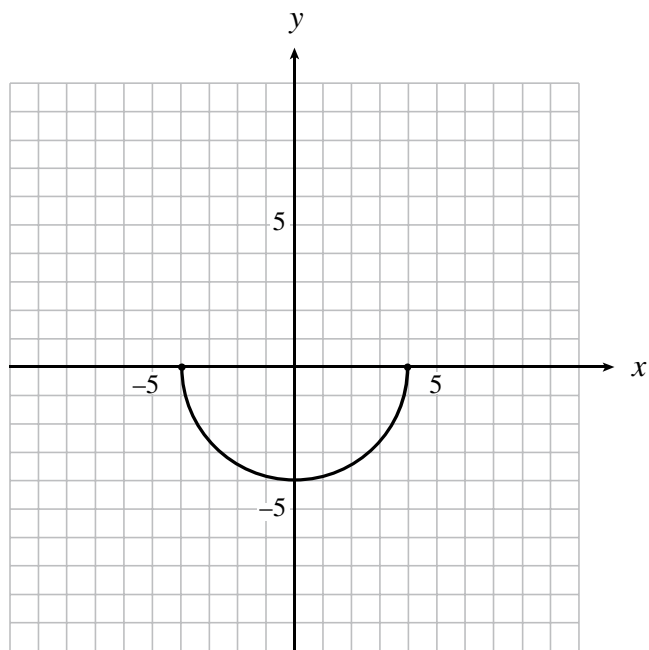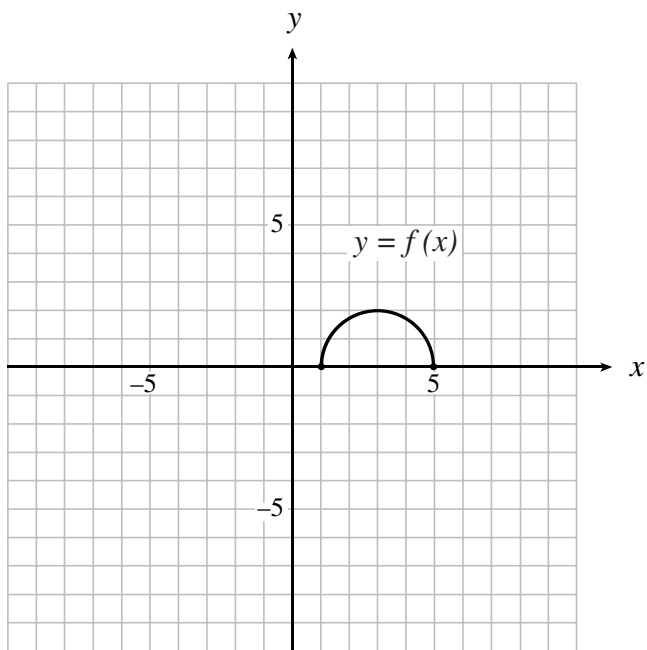A. $y = 2f(x+1)$
B. $y = f(2x-1)$
C. $y = f(2x+1)$
D. $y = 2f(x-1)$

23. The graph of $y = f(x)$ is shown below. On the grid provided, sketch the graph of $y = -f\left(\frac{1}{2}(x+2)\right)$.

24. The graph of $y = f(x)$ is shown below on the left. Which equation represents the graph shown on the right?

A. $y = -2f(2x + 3)$

B. $y = -2f(2x + 6)$

C. $y = -2f\left(\frac{1}{2}x + 3\right)$

D. $y = -2f\left(\frac{1}{2}x + 6\right)$

25. If the point $(6, -2)$ is on the graph $y = f(x)$, which point must be on

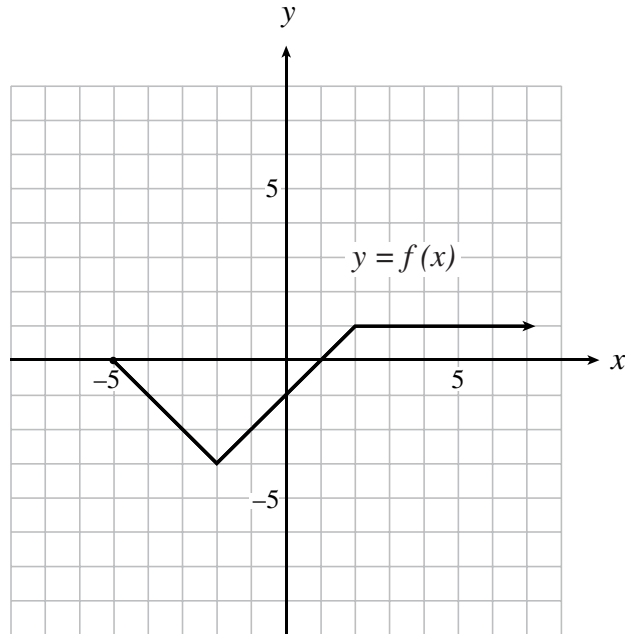the graph of $y = \dfrac{1}{f(-x)+4}$?

A. $\left(-10, -\dfrac{1}{2}\right)$

B. $\left(-6, \dfrac{1}{2}\right)$

C. $\left(-6, \dfrac{7}{2}\right)$

D. $\left(-\dfrac{1}{6}, 2\right)$

26. The graph of $y = f(x)$ is shown below.

y

$y = f(x)$

x

5

−5

5

−5

Understanding
B6

a) On the grid provided, sketch the graph of:
$$y = 2\left|f(x)\right| + 1$$

b) On the grid provided, sketch the graph of:
$$y = 2\left|f(x) + 1\right|$$